# Economics of Innovation

Lecture 2 -
Endogenous economic growth theory

http://www.economics-of-innovation.com



Erasmus University Rotterdam
Department of Applied Economics
Dr. Philipp Koellinger

## Questions addressed by growth theory

- Why do economies grow?
- Why are the people in some countries poor, but rich in others?
- Can the wealth of nations ever converge? Or do the poor stay poor, while the rich get richer?
- Can economic growth continue forever? Or are there limits to growth?

→ "Once you start thinking about growth, it is difficult to think about anything else." (Economist Robert Lucas, Nobel laureate 1995)



Erasmus University Rotterdam
Department of Applied Economics
Dr. Philipp Koellinger

## The insights of the Solow growth model

- Output per worker and capital per worker converge to particular *steady state* values in the long run
- Once this *steady state* has been reached, *technological progress* is the only factor that can further increase the output per worker (and hence average individual wealth)
- Countries that are structurally identical converge to the same level of wealth
  - Savings rate (+)
  - Technological development (+)
  - Population growth rate (-)
  - Depreciation rate of physical capital (-)
  - Income share of capital and labor (capital +)

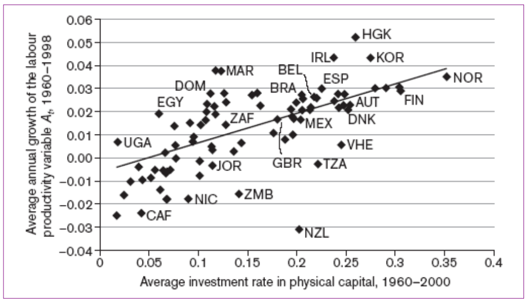
## Limitations of the Solow growth model

- Technological progress is exogenous, it falls as "manna from heaven" (thus also called *exogenous growth model*)
  - How can technological progress be facilitated?
  - Is technological progress and hence economic growth sustainable forever?
- The model's forecast of conditional economic convergence among countries are too optimistic
- The model cannot explain why we did not observe any significant economic growth before 1800
- The model cannot explain the empirically significant positive relationship between saving rates and technological progress (see next slide)





Average annual rate of labour-augmenting technological progress against average investment rate in physical capital, 84 countries




Sources: Penn World Table 6.1 and data set for Bernanke and Gürkaynak (2001), see Sørensen and Whitta-Jacobsen 2005, p. 239.





## Towards endogenous growth theory

- Endogenous growth theory
  - Explains specifically where technological change comes from
  - How it can be influenced by behavioral variables
  - Explains under which conditions economic growth continues forever, or eventually slows down and stagnates at some point
- Basic assumptions
  - Exogenous shocks do not occur
  - Expectations are correct
  - Prices and quantities are fully adjusted by perfect competition in final goods and labor markets
  - Monopoly rents for new ideas can be earned because each valuable new idea is assumed to be fully protected by an eternal patent (new additional assumption of endogenous growth theory)

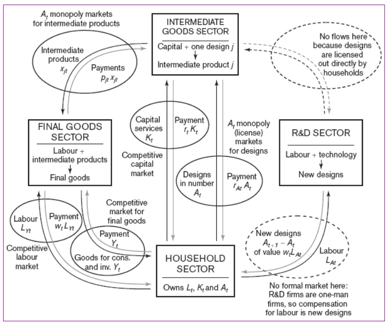
## Additional assumptions

- Labor, capital and technology are the only production factors
  - Entrepreneurship, education and possibly exhaustive natural resources are not explicitly included here
- All new ideas get used and are worth the same
- Existing technology promotes the production of new ideas
  - “Standing on shoulders” effect, e.g. computers, mathematics, production techniques… Unlimited number of ideas to be discovered (φ=1)
  - No “stepping on toes” (λ=1)



Erasmus University Rotterdam
Department of Applied Economics
Dr. Philipp Koellinger

## Flow chart of a Romer economy






Erasmus University Rotterdam
Department of Applied Economics
Dr. Philipp Koellinger

## Model of endogenous growth - 1

- Aggregate values

1) $Y_t = K_t^{\alpha}(A_t L_{Yt})^{1-\alpha}$

$Y$ *is output*, $A$ *is technol. level*, $L$ *is labour*, $K$ *is capital*

2) $A_{t+1} - A_t = \rho A_t L_{At}$ $\quad \rho$ *is productivity of researchers*

3) $K_{t+1} = sY_t + (1-\delta)K_t$

$s$ *is savings rate*, $\delta$ *is rate of depreciation*

4) $L_{Yt} + L_{At} = L$

*Labour restriction*

$0 < \alpha, \delta, s < 1$

$t = 0, 1, 2, \ldots$

$\rho \geq 0$

## Comments on the production function

- Each final-goods firm must take the existing stock of technological knowledge as given, they cannot adjust it.
- The available stock of knowledge (i.e. all intermediate goods) can be used in *every* firm, it is non-rival.
- Production has constant returns to the two rival inputs:
  $f(\lambda L, \lambda K) = \lambda f(L, K)$
- But the entire production function has increasing returns because of the nonrival input factor:
  $f(\lambda A, \lambda L, \lambda K) > \lambda f(A, L, K)$





## Model of endogenous growth - 2

- Endogenous determination of R&D share:
  $w_t = (1-\alpha)\frac{Y_t}{L_{Yt}}$ $w$ is real wage rate
  $r_t = \alpha^2 \frac{Y_t}{K_t}$ $r$ is rental rate for one unit of capital $(K)$
  $\pi_t = \alpha(1-\alpha)\frac{Y_t}{A_t}$ $\pi$ is profit for one patent / idea $(A)$
- These are equilibrium prices that enter the endogenous determination of the R&D share
- They result from the basic assumptions on slide 6





## Model of endogenous growth - 3

- Endogenous determination of R&D share continued:
  $(r_t - \delta)P_{At} = \pi_t + P_{At+1} - P_{At}$ where $P_{At}$ is the amount of capital that would be just as good as holding a patent in period $t$, and $\delta$ is the depreciation rate of capital
  $w_t = \rho A_t P_{At+1}$ where $\rho$ indicates the productivity of researchers
  $L_{At} = s_{Rt} L$ where $s_{Rt}$ is the share of employees working in R & D
  $0 < s_{Rt} < 1$
- Remark - in steady-state equilibrium:
  $P_{At+1} = P_{At} = P_A$ for all $t$

## Balanced growth path

- A balanced growth path of the model must fulfill the following requirements:
  - A series of $A_t$, $K_t$ and $L_t$ for which all previously specified model equations hold
  - A constant capital-output ratio over time, $K / Y$
  - A constant interest rate over time, $(r - \delta)$
  - Capital per worker ($K / L$), output per worker ($Y / L$), and the wage rate ($w$) all growing at one and the same constant rate
  - These last three requirements are imposed by the empirically observable growth patterns across countries and time
- Strategy: Assume that $s_{rt}$ is constant and equal to $s_r$ in all periods and show that the model equations imply a balanced growth path





## Balanced growth aggregate values

- Assuming a constant research share $s_r$:

$$Y_t = K_t^{\alpha}(A_t(1-s_r)L)^{1-\alpha}$$
$$K_{t+1} = sY_t + (1-\delta)K_t$$
$$A_{t+1} = (1+g_e)A_t$$

- Similarly to the basic Solow model, we can define technology-adjusted variables for income per worker and capital per worker that will remain constant over time (for the so-called steady-state prediction of the model)

$$\tilde{k} \equiv K_t / A_t L_t = k_t / A_t$$
$$\tilde{y} \equiv Y_t / A_t L_t = y_t / A_t$$





## Steady-state predictions - 1

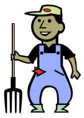
- The model converges to a steady state for $\tilde{k}^*$ and $\tilde{y}^*$
- Output per worker and capital per worker both grow at the constant rate of $g_e$ (rate of technological progress), and the capital-output ratio is constant over time:

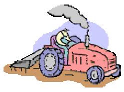
$$\tilde{k}^* = \left(\frac{s}{g_e + \delta}\right)^{1/1-\alpha}(1-s_r)$$
$$\tilde{y}^* = \left(\frac{s}{g_e + \delta}\right)^{\alpha/1-\alpha}(1-s_r)$$
$$\frac{K_t}{Y_t} = \frac{\tilde{k}^*}{\tilde{y}^*} = \frac{s}{g_e + \delta}$$

## Steady-state growth

1950

2000





$$y^*_{1950} = \frac{Y^*_{1950}}{L_{1950}} < y^*_{2000} = \frac{Y^*_{2000}}{L_{2000}}$$

$$k^*_{1950} = \frac{K^*_{1950}}{L_{1950}} < k^*_{2000} = \frac{K^*_{2000}}{L_{2000}}$$

$$\tilde{k}^* = \frac{K^*_t}{A_t L_t} = \frac{K^*_{t+1}}{A_{t+1} L_{t+1}}$$

$$\tilde{y}^* = \frac{Y^*_t}{A_t L_t} = \frac{Y^*_{t+1}}{A_{t+1} L_{t+1}}$$

- GDP per capita and capital intensity increase both at the rate of technological progress, *g*
- Technological progress makes both capital and labor more productive
- In the steady state, technology-adjusted capital intensity $\tilde{k}^*$ and $\tilde{y}^*$ technology-adjusted output per worker remain constant





## Steady-state predictions - 2

- Wages increase at the rate of technological progress, while the interest rate and the rental rate for designs remain constant:

$$w_t = \frac{(1-\alpha)}{(1-s_R)} \tilde{y}^* A_t$$

$$r_t = r \equiv \alpha^2 \frac{g_e + \delta}{s}$$

$$\pi_t = \alpha(1-\alpha)\tilde{y}^* L$$

- All these results imply a balanced growth path of the model for any constant value $0 < s_R < 1$





## Intermediate step

- Reconsider the arbitrage condition:

$$w_t = \rho A_t P_{At+1}$$

- Since $w_t$ and $A_t$ grow at the common rate, $g_e$:

$$P_{At+1} = P_{At} = P_A \ \textit{for all } t$$

- The arbitrage condition thus becomes

$$P_A = \frac{w_t}{\rho A_t} = \frac{1}{\rho} \frac{1-\alpha}{1-s_R} \tilde{y}^*$$

- Since the capital value of patents is constant in the steady state:

$$P_A = \frac{\pi}{r-\delta} = \frac{\alpha(1-\alpha)}{r-\delta} \tilde{y}^* L$$

## Steady-state predictions - 3

- Thus, the capital value of patents can be expressed in two ways:

$$P_A = \frac{1}{\rho}\frac{1-\alpha}{1-s_R}\tilde{y}^* = \frac{\alpha(1-\alpha)}{r-\delta}\tilde{y}^* L$$

- Equating both expressions and solving for $s_R$ yields the endogenous value of the research share:

$$s_R^* = \frac{1+\left(1-\frac{\alpha^2}{s}\right)\frac{\delta}{\alpha\rho L}}{1+\frac{\alpha}{s}}$$



Erasmus University Rotterdam
Department of Applied Economics
Dr. Philipp Koellinger

## Main messages

- The rate of technological progress is endogenously determined:

$$g_e^* = \rho s_R L = \frac{s\delta + Ls\alpha\rho - \alpha^2\delta}{s\alpha + \alpha^2}$$

- It depends on
  - *The productivity of workers in the research sector (+)*
  - *The size of the economy / the number of workers (+)*
  - *The savings rate (+)*
  - The capital share of income (-)
  - The depreciation rate of capital (+ if $s > \alpha^2$ )
- This model suggests eternally sustainable economic growth



Erasmus University Rotterdam
Department of Applied Economics
Dr. Philipp Koellinger

## Further insights on the model's conclusions

- Positive effect of productivity of R&D workers
  - Might be stimulated by superior education and research resources (e.g. Internet, digital libraries, big research centers)
- Positive effect of L
  - Implies more workers in R&D and therefore more new ideas
  - But stimulating population growth is not a good strategy (empirical evidence speaks strongly against this)
  - More refined interpretation: Available human capital increases because of free trade, mobility of products, workers, and ideas. Globalization increases R&D productivity (Romer 1990)

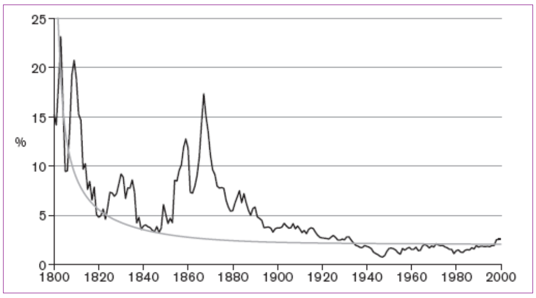
# Further insights on the model's conclusions

- Positive effect of the savings rate
  - More savings imply more capital
  - More capital implies lower marginal returns to capital and hence lower rental rates for capital
  - Thus, lower interest rate and higher present value of patents
  - This higher payoff for patents makes research more attractive as an income opportunity and increases the labor share in research and the output of new ideas
- Positive effect of the depreciation rate
  - Also implies lower interest rate and thus higher present value for patents - recall that the interest rate is $(r_t - \delta)$
- Negative effect of capital share of income
  - Implies lower wages and hence in equilibrium less R&D





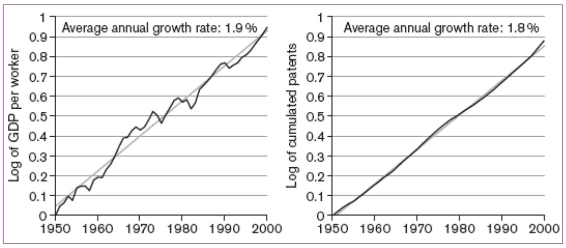
# Annual growth rate of the cumulative number of patents granted in the USA








# Endogenous growth - Log of GDP per worker (left) and log of cumulated patents (right) for the USA



## Is growth truly endogenous?

- If yes, economic growth will continue forever.
- Crucial parameters:
  - Constant returns to R&D (φ=1), "standing on shoulders", no "fishing out" of ideas or creative destruction
  - No "stepping on toes" (λ=1)
- Alternative 1: Semi-endogenous growth
  - 0 < φ < 1, decreasing returns to R&D production, it becomes harder and harder to discover something new
  - 0 < λ < 1, negative externality of R&D production possible
  - Result: Growth cannot continue forever, it can only be accomplished by increases in L (population size, human capital)





## Is growth truly endogenous?

- Alternative 2: Hemi-endogenous growth (Jones 2002)
  - 0 < φ < 1
  - An increasing share of labor in the R&D sector is required to maintain growth
  - Increases in $s_r$ shift the steady state to higher level, adjustment towards new steady state is slow
  - Constant, small increases in $s_r$ are compatible with (convergent) growth over long time periods
  - Story fits data from the US for the last 200 years
  - Eventually, growth would stop (limits to $s_r$), but we would have reached a very high level of material wealth then





## Is growth truly endogenous?

- The final verdict is still out…
  - Madsen (2007): The null-hypotheses of constant returns to R&D production (φ = 1) cannot be rejected for 21 OECD countries 1965-2004
  - Jones (2009, 2010): Inventors reach their productivity peaks later and later – one needs more inventors now than earlier
  - Can we find perfect substitutes for scarce natural resources (oil, coal…)?
  - Can we keep population growth under control?
- Debate continues…
- …but even the most pessimistic interpretations suggest that growth can continue for quite some time

## Policy conclusions from Romer model

- Long run growth potential can be positively influenced by
  - Government investments in education (improves human capital)
  - Government investments in R&D
    - Subsidies to R&D
    - Public R&D centers and universities
  - Private incentives to conduct R&D (e.g. patent rights and copyright protection)
  - Globalization, trade, and cross-border mobility of knowledge
  - Increases in the savings rate
    - Trade-off with consumption
    - “Golden rule” of savings